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Study of Butterfly Patterns of Matrix in Interconnection Network

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Abstract

We have presented the unique patterns obtained from Sparse matrices of interconnected networks. We showed properties derived from these patterns. We have presented the common properties obtained from these patterns. We have showed that matrices obtained are alpha-beta band matrices. The routing function is evaluated from alpha-beta band matrices.

Keywords: Sparse Matrix, Multicore Architecture, Hypercube, Adjacency Matrix.

Introduction:

In this research, we have given a method to obtain Sparse Matrix of some interconnected networks. Sparse matrices have most of its elements as null, so this makes study of it easy. Since each interconnection network has its own properties, these properties are visible in their respective matrix. We used sparse matrix to describe properties of interconnection network. We have proved these properties by using concepts of matrix. We have presented the unique patterns obtained from sparse matrices. These patterns are indifferent and are related to their network.

Background:

We have considered Sparse Matrix to study patterns of interconnection network. A sparse Matrix is a two-dimensional array having the value of majority elements as null. [12] Following is a sparse matrix where '*' denotes the elements having non-null values.

$$\begin{bmatrix} \_ & * & * & * & * \\ * & \_ & * & \_ & * \\ * & * & \_ & * & * \\ * & \_ & * & \_ & * \\ * & * & * & * & \_ \end{bmatrix}$$

Figure 1. A Sparse Matrix

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In large number of applications, sparse matrices are involved. Some well-known sparse matrices which are symmetric in form can be classified as follows:

1. Lower- left Triangular Matrices- Triangular Matrices which has elements in its lower left part only are called Lower-left triangular matrices.

2. Lower right Triangular Matrices- Triangular Matrices which has elements in its lower right part only are called Lower-left triangular matrices.

3. Upper- left Triangular Matrices- Triangular Matrices which has elements in its upper left part only are called Lower-left triangular matrices.

4. Upper- right Triangular Matrices- Triangular Matrices which has elements in its upper right part only are called Lower-left triangular matrices.

5. Diagonal Matrices- Matrices which has elements at its diagonal only are called Diagonal matrices.

6. Tridiagonal Matrices-Matrices which has elements at its diagonal as well as at upper and lower parts of diagonal as shown below in Figure 4.

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Figure 2. Tridiagonal Matrix

7. αβ-Band Matrices- Matrices which has elements at the upper and lower part of the diagonal as shown below in Figure 5 are called αβ-Band Matrices.

$$\begin{array}{cc} \alpha & \\ \begin{bmatrix} 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \end{bmatrix} & \beta \end{array}$$

Figure 3. αβ-Band Matrix

When we obtained Adjacency matrices of network, we got most of them are of αβ-Band matrices type. So, the indexing formula of αβ-Band Matrix can be used as routing function for those architectures which has αβ-Band Matrix pattern. [7] Considering the row-major ordering for the memory allocation, the indexing formula is explained as below: [12]

Case 1: $1 \leq i \leq \beta$

Address (a_{ij}) = Number of elements in first (i-1)-th rows + Number of elements in i-th row up to j-th columns

$$= \alpha + (\alpha + 1) + (\alpha + 2) + \cdots + (\alpha + i - 2) + j$$
$$= \alpha \times (i - 1) + [1 + 2 + 3 + \cdots + (i - 2)] + j$$
$$= \alpha \times (i - 1) + \frac{(i - 1)(i - 2)}{2} + j$$

Case 2: β<i≤n-α+1

Address (aij) = Number of first β rows+ Number of elements between (β+1)-th row and (i-1)-th row + Number of elements in i-th row

$$= \alpha + (\alpha + 1) + (\alpha + 2) + \cdots + (\alpha + \beta - 1) + (\alpha + \beta - 1) \times (i - \beta - 1) + j - i + \beta$$

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$$= \alpha\beta + \frac{\beta(\beta-1)}{2} + (\alpha\alpha + \beta\beta - 1)(i - \beta - 1) + j - i + \beta$$

Case 3: n-α+1<i

Address (aij) = Number of elements in first (n-α+1) rows + Number of elements after (n-α+1)-th row and up to (i-1)-th row + Number of elements in i-th row

$$= \alpha\beta + \frac{\beta(\beta-1)}{2} + (\alpha+\beta-1)(n-\alpha-\beta+1) + (\alpha+\beta-2)$$

$$+(\alpha+\beta-3) + \cdots + \{\alpha+\beta - [(i-1)-(n-\alpha+1)]\} + j - i + \alpha$$

$$= \alpha\beta + \frac{\beta(\beta-1)}{2} + (\alpha+\beta-1)(n-\alpha-\beta+1) + (\alpha+\beta)(i-n+\alpha-1) - \frac{(i-n+\alpha-1)+(i-n+\alpha-2)}{2} + 1$$

Matrix Representation of Some Interconnection Network:

1. Mesh

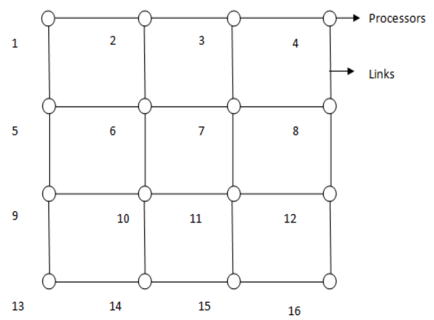

Figure 4. Graphical Representation of Mesh

$$\begin{bmatrix}
0&1&0&0&1&0&0&0&0&0&0&0&0&0&0&0\\
1&0&1&0&0&1&0&0&0&0&0&0&0&0&0&0\\
0&1&0&1&0&0&1&0&0&0&0&0&0&0&0&0\\
0&0&1&0&0&0&0&1&0&0&0&0&0&0&0&0\\
1&0&0&0&0&1&0&0&1&0&0&0&0&0&0&0\\
0&1&0&0&1&0&1&0&0&1&0&0&0&0&0&0\\
0&0&1&0&0&1&0&1&0&0&1&0&0&0&0&0\\
0&0&0&1&0&0&1&0&0&0&0&1&0&0&0&0\\
0&0&0&0&1&0&0&0&0&1&0&0&1&0&0&0\\
0&0&0&0&0&1&0&0&1&0&1&0&0&1&0&0\\
0&0&0&0&0&0&1&0&0&1&0&1&0&0&1&0\\
0&0&0&0&0&0&0&1&0&0&1&0&0&0&0&1\\
0&0&0&0&0&0&0&0&1&0&0&0&0&1&0&0\\
0&0&0&0&0&0&0&0&0&1&0&0&1&0&1&0\\
0&0&0&0&0&0&0&0&0&0&1&0&0&1&0&1\\
0&0&0&0&0&0&0&0&0&0&0&1&0&0&1&0
\end{bmatrix}$$

Figure 5. Incidence Matrix of Mesh

i\j	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0
2	1	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0
3	0	1	0	1	0	0	1	0	0	0	0	0	0	0	0	0
4	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0
5	1	0	0	0	0	1	0	0	1	0	0	0	0	0	0	0
6	0	1	0	0	1	0	1	0	0	1	0	0	0	0	0	0
7	0	0	1	0	0	1	0	1	0	0	1	0	0	0	0	0
8	0	0	0	1	0	0	1	0	0	0	0	1	0	0	0	0
9	0	0	0	0	1	0	0	0	0	1	0	0	1	0	0	0
10	0	0	0	0	0	1	0	0	1	0	1	0	0	1	0	0
11	0	0	0	0	0	0	1	0	0	1	0	1	0	0	1	0
12	0	0	0	0	0	0	0	1	0	0	1	0	0	0	0	1
13	0	0	0	0	0	0	0	0	1	0	0	0	0	1	0	0
14	0	0	0	0	0	0	0	0	0	1	0	0	1	0	1	0
15	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	1
16	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0

Figure 6. Butterfly Pattern of Mesh

2. Pyramid

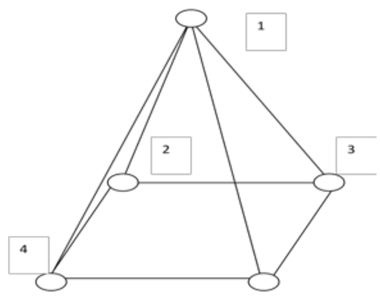

Figure 7. Graphical Representation of Pyramid

$$\begin{bmatrix}
0&1&1&1&1\\
1&0&1&1&0\\
1&1&0&0&1\\
1&1&0&0&1\\
1&0&1&1&0
\end{bmatrix}$$

Figure 8. incidence Matrix of Pyramid

i\j	1	2	3	4	5
1	0	1	1	1	1
2	1	0	1	1	0
3	1	1	0	0	1
4	1	1	0	0	1
5	1	0	1	1	0

Figure 9. Butterfly Pattern of Pyramid

3. Torus

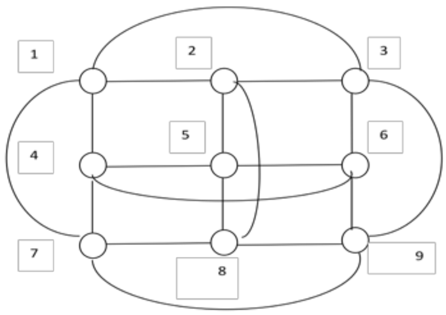

Figure 10. Graphical Representation of Torus

$$\begin{bmatrix} 0 & 1 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

Figure 11. Incidence Matrix of Torus

i\j	1	2	3	4	5	6	7	8	9
1	0	1	1	1	0	0	1	0	0
2	1	0	1	0	1	0	0	1	0
3	1	1	0	0	0	1	0	0	1
4	1	0	0	0	1	1	1	0	0
5	0	1	0	1	0	1	0	1	0
6	0	0	1	1	1	0	0	0	1
7	1	0	0	1	0	0	0	1	1
8	0	1	0	0	1	0	1	0	1
9	0	0	1	0	0	1	1	1	0

Figure 12. Butterfly Pattern of Torus

4. Hypercube

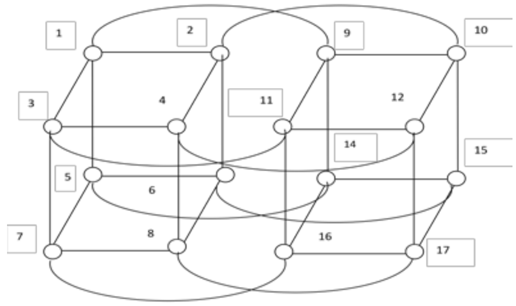

Figure 13. Graphical Representation of Hypercube

$$\begin{bmatrix} 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \end{bmatrix}$$

Figure 14. Incidence Matrix of Hypercube

i\j	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	0	1	1	0	1	0	0	0	1	0	0	0	0	0	0	0
2	1	0	0	1	0	1	0	0	0	1	0	0	0	0	0	0
3	1	0	0	1	0	0	1	0	0	0	1	0	0	0	0	0
4	0	1	1	0	0	0	0	1	0	0	0	1	0	0	0	0
5	1	0	0	0	0	1	1	0	0	0	0	0	1	0	0	0
6	0	1	0	0	1	0	0	1	0	0	0	0	0	1	0	0
7	0	0	1	0	1	0	0	1	0	0	0	0	0	0	1	0
8	0	0	0	1	0	1	1	0	0	0	0	0	0	0	0	1
9	1	0	0	0	0	0	0	0	0	1	1	0	1	0	0	0
10	0	1	0	0	0	0	0	0	1	0	0	1	0	1	0	0
11	0	0	1	0	0	0	0	0	1	0	0	1	0	0	1	0
12	0	0	0	1	0	0	0	0	0	1	1	0	0	0	0	1
13	0	0	0	0	1	0	0	0	1	0	0	0	0	1	1	0
14	0	0	0	0	0	1	0	0	0	1	0	0	1	0	0	1
15	0	0	0	0	0	0	1	0	0	0	1	0	1	0	0	1
16	0	0	0	0	0	0	0	1	0	0	0	1	0	1	1	0

Figure 15. Butterfly Pattern of Hypercube

5. Butterfly

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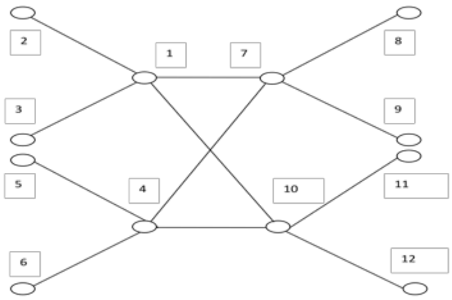

Figure 16. Graphical Representation of Butterfly

$$\begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

Figure 17. Incidence Matrix of Butterfly

i\j	1	2	3	4	5	6	7	8	9	10	11	12
1	0	1	1	0	0	0	1	0	0	1	0	0
2	1	0	0	0	0	0	0	0	0	0	0	0
3	1	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	1	1	1	0	0	1	0	0
5	0	0	0	1	0	0	0	0	0	0	0	0
6	0	0	0	1	0	0	0	0	0	0	0	0
7	1	0	0	1	0	0	0	1	1	0	0	0
8	0	0	0	0	0	0	1	0	0	0	0	0
9	0	0	0	0	0	0	1	0	0	0	0	0
10	1	0	0	1	0	0	0	0	0	0	1	1
11	0	0	0	0	0	0	0	0	0	1	0	0
12	0	0	0	0	0	0	0	0	0	1	0	0

Figure 18. Butterfly Pattern

6. Fat Tree

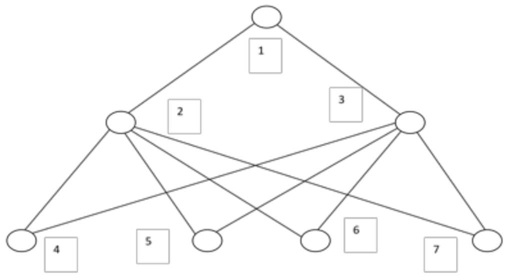

Figure 19. Graphical Representation of Fat tree

$$\begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Figure 20. Incidence Matrix of Fat Tree

i\j	1	2	3	4	5	6	7
1	0	1	1	0	0	0	0
2	1	0	0	1	1	1	1
3	1	0	0	1	1	1	1
4	0	1	1	0	0	0	0
5	0	1	1	0	0	0	0
6	0	1	1	0	0	0	0
7	0	1	1	0	0	0	0

Figure 21. Butterfly Pattern of Fat Tree

7. Perfect Difference Network

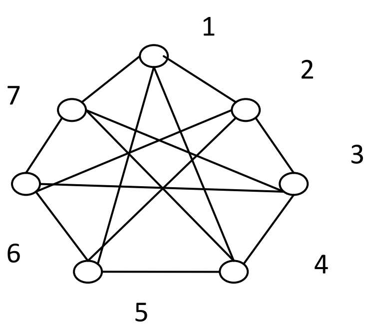

Figure 22. Graphical Representation of PDN

$$\begin{bmatrix} 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 \end{bmatrix}$$

Figure 23. Incidence Matrix of PDN

i/j	1	2	3	4	5	6	7
1	0	1	0	1	1	0	1
2	1	0	1	0	1	1	0
3	0	1	0	1	0	1	1
4	1	0	1	0	1	0	1
5	1	1	0	1	0	1	0
6	0	1	1	0	1	0	1
7	1	0	1	1	0	1	0

Figure 24. Butterfly Pattern of PDN

List of Properties obtained:

1. The above Matrices so obtained are all Sparse Matrices (say A).

2. Transpose of A is equal to itself. That is, $A=A^T$

3. In some, upper right triangular part of A is symmetrical to its lower left triangular part. Similarly, in some upper left triangular part is symmetrical to its right lower triangular part.

4. On bisecting the A horizontally from the center, the upper part is symmetrical to lower part but is in opposite direction.

5. On bisecting the A vertically from the center, the left part is symmetrical to right part but some are in opposite direction and some are in same direction.

6. 1 at the intersection of a row and a column denotes connection, i.e., node of that row is connected with the node of that column. 0 at the intersection of a row and a column denotes that they do not have a connection with each other.

7. Number of edges or links in the architecture is the total number of 1s in the matrix.

8. Degree of a node (represented by either a row or column) is equal to the total number of 1s in the row or column.

9. The Routing Function of it can be formulated by alpha- beta band matrix.

10. On observing matrices of each network, we see that each have a unique pattern.

Conclusion

When we have derived the matrix patterns of architectures, we observed that all matrices are Sparse Matrices i.e., the connections between processors are loose and not dense. This concludes that the communication is also loose between two nodes. The unique patterns so observed conclude that each interconnection network has its own unique property. These matrices may help in recognizing the architectures and in study of its properties.

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